Energy-Efficient Transmission Scheduling for Wireless Media Streaming with Strict Underflow Constraints

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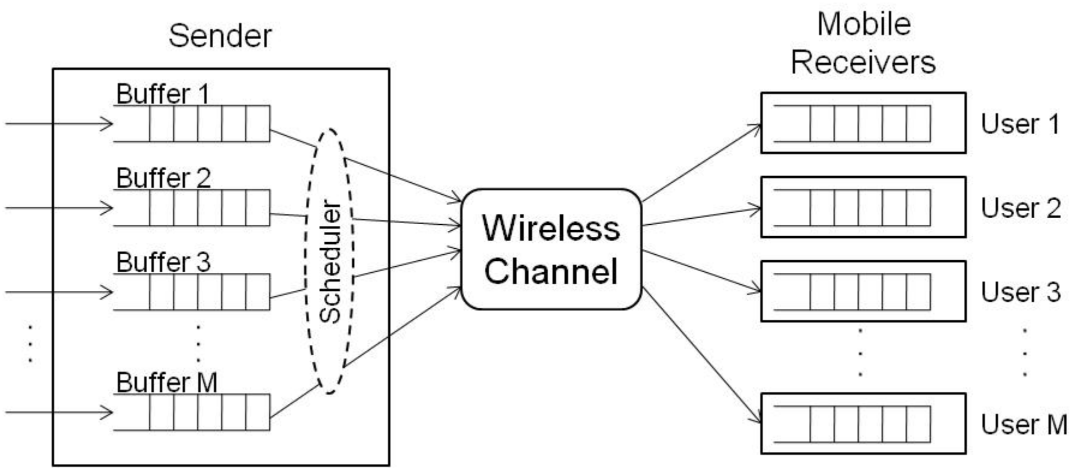
Wireless Media Streaming



Key Features

- Single source transmitting media streams to multiple users over a shared wireless channel
- Available data rate of the channel varies with time and from user to user

Desirable Operating Characteristics

- Provide high playout quality for all users
- Maximize the number of users that can be supported
- Limit interference to other senders and their associated mobiles
- Extend system lifetime (when sender is a mobile device)

Opportunistic Scheduling

- Exploit temporal and spatial variation of the channel by transmitting to users with the best available data rates
  - “Multiuser diversity gain,” introduced in context of analogous *uplink* problem

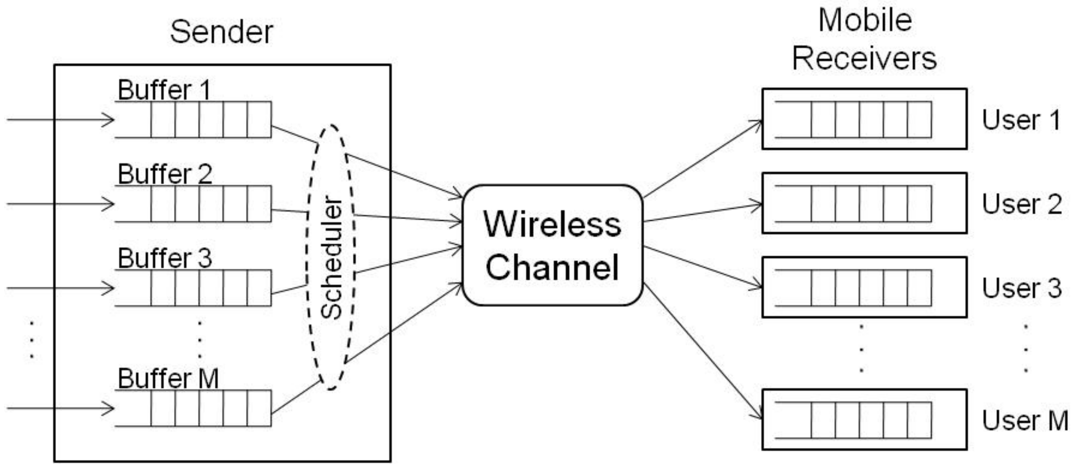
- Consider a notion of fairness
  - Temporal fairness
  - Proportional fairness
  - Utilitarian fairness
  - ...

- Our approach
  - Playout quality is closely linked to receiver buffer underflow
  - Accordingly, we introduce strict buffer underflow constraints as our notion of fairness

Outline

- **Problem Description and Formulation**
- Single User Case
- Multiple Users Case
- Future Work and Concluding Remarks

Problem Description



Timing in Each Slot

- Transmitter learns each channel's state through a feedback channel
- Transmitter allocates some amount of power (possibly zero) for transmission to each user
  - Total power allocated in any slot cannot exceed a power constraint, *P*
- Transmission and reception
- Packets removed/purged from each receiver's buffer for playing

Problem Description (cont.)

Two Control Objectives

- Avoid underflow, so as to maintain playout quality
- Minimize system-wide power consumption

Key Modeling Assumptions

- Sender always has data to transmit to each receiver
- Receivers have infinite buffers
- Slot duration within channel coherence time (condition constant over slot)
- Each user's channel condition is *i.i.d.*
- Transmitter knows precisely the packet requirements of each user in each time slot (as it knows the encoding and decoding schemes)
- Each user's per slot consumption of packets is constant over time, d^m
- Packets transmitted during a slot arrive in time to be played in the same slot
- The available power P is always sufficient to transmit packets to cover one slot of playout for each user

Finite and Infinite Horizon Problem Formulation
Cost Structure, Information State, and Action Space

Cost Structure

- Transmission costs
  - Assume a linear power-rate function
  - C_n^m is a random variable describing power consumption per unit of data transmitted to user *m* at time *n* (including retransmissions)
  - Realizations of C_n^m lie in $(0, c_{MAX}^m]$
- Holding costs
  - Per packet per slot holding cost h^m assessed on all packets remaining in user *m*'s receiver buffer after playout consumption
  - Technical assumption – can take h^m arbitrarily small

Information State

- $X_n = \left(X_n^1, X_n^2, \ldots, X_n^M\right)^T$ = vector of receiver buffer queue lengths at time *n*
- $C_n = \left(C_n^1, C_n^2, \ldots, C_n^M\right)^T$ = vector of channel conditions for slot *n*

Action Space

- Defined in terms of ***y***, receiver buffer queue levels ***after*** transmission
- Must satisfy strict underflow constraints and system-wide power constraint
- $\mathcal{A}(\mathbf{x}, \mathbf{c}) := \left\{ \mathbf{y} \in \mathbb{R}_+^M : \max(\mathbf{x}, \mathbf{d}) \preceq \mathbf{y} \text{ and } \mathbf{c}^T(\mathbf{y} - \mathbf{x}) \leq P \right\}$

Finite and Infinite Horizon Problem Formulation
System Dynamics, Optimization Criteria, and Optimization Problems

System Dynamics

- $\mathbf{X}_{n-1} = \mathbf{Y}_n - \mathbf{d}$
- ***C****$_{n-1}$* generated as *i.i.d.* random variable

Optimization Criteria

- Finite horizon expected discounted cost criterion:

$$J_N^\pi := \mathbb{E}^\pi \left\{ \sum_{m=1}^{M} \sum_{t=1}^{N} \alpha^{N-t} \cdot \left\{ C^m \cdot (Y_t^m - X_t^m) + h^m \cdot (Y_t^m - d^m) \right\} \mid \mathcal{F}_N \right\}$$

- Infinite horizon expected discounted cost criterion:

$$J_\infty^\pi = \lim_{N \to \infty} J_N^\pi$$

Optimization Problems

$$\min_{\pi \in \Pi} J_N^\pi \quad \left(\text{or} \quad \min_{\pi \in \Pi} J_\infty^\pi \right)$$

$$\text{s.t.} \quad \sum_{m=1}^{M} C_n^m \cdot (Y_n^m - X_n^m) \le P, \quad \forall n \quad \text{and}$$

$$Y_n^m \ge \max(X_n^m, d^m), \quad \forall m, \forall n.$$

Relation to Inventory Theory

- In inventory language, our problem is a multi-period, multi-item, discrete time inventory model with random ordering prices, deterministic demand, and a budget constraint
  - Items / goods → mobile receivers
  - Random ordering prices → random channel conditions
  - Deterministic demand → users' packet requirements for playout
  - Budget constraint → transmitter's power constraint

- To our knowledge, this model has not been studied, but there is some related work
  - Single item inventory models with random ordering prices
    - B. G. Kingsman, 1969
    - K. Golabi, 1985
  - Single and multiple item inventory models with stochastic demands and deterministic ordering prices
    - R. Evans, 1967
    - A. Federgruen and P. Zipkin, 1986
    - G. A. DeCroix, 1998

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Single User Case
Finite Horizon Problem

Dynamic Program

$$
\begin{aligned}
V_n(x,c) &= \min_{\max(x,d)\le y\le x+\frac{P}{c}} \left\{ c\cdot(y-x) + h\cdot(y-d) + \alpha\cdot \mathbb{E}\left[V_{n-1}(y-d,C)\right] \right\} \\
V_0(x,c) &= 0, \quad \forall x\in \mathbb{R}_+, \forall c\in \mathcal{C} = [c_{\min}, c_{\max}]
\end{aligned}
$$

- Uncountable state space and uncountable action space
- Computationally intractable

Single User Case
Finite Horizon Problem

Equivalent Dynamic Programming Equation

$$
\begin{aligned}
V_n(x,c) &= \min_{\max(x,d)\le y\le x+\frac{P}{c}} \left\{ c\cdot(y-x) + h\cdot(y-d) + \alpha\cdot \mathbb{E}\big[V_{n-1}(y-d,C)\big] \right\} \\
&= -c\cdot x - h\cdot d + \min_{\max(x,d)\le y\le x+\frac{P}{c}} \left\{ y\cdot(c+h) + \alpha\cdot \mathbb{E}\big[V_{n-1}(y-d,C)\big] \right\} \\
&= -c\cdot x - h\cdot d + \min_{\max(x,d)\le y\le x+\frac{P}{c}} \left\{ g_n(y,c) \right\}
\end{aligned}
$$

- If action space were independent of *x*, we would have a base-stock policy
- Instead, we get a modified base-stock policy

Single User Case
Structure of Optimal Policy

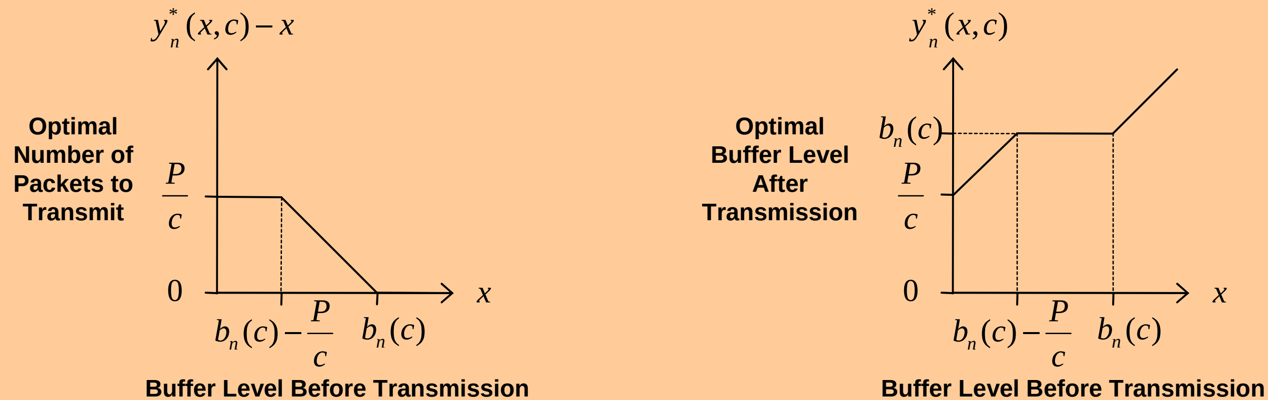
Theorem

For every $n \in \{1,2,\ldots,N\}$ and $c \in \mathcal{C}$, there exists a *critical number*, $b_n(c)$, such that the optimal control strategy is given by $\pi^* = \{y_N^*, y_{N-1}^*, \cdots, y_1^*\}$, where

$$y_n^*(x,c) := \begin{cases} x, & \text{if } x \ge b_n(c) \\ b_n(c), & \text{if } b_n(c) - \frac{P}{c} \le x \le b_n(c) \\ x + \frac{P}{c}, & \text{if } x < b_n(c) - \frac{P}{c} \end{cases}.$$

Furthermore, for a fixed n, $b_n(c)$ is nonincreasing in c, and for a fixed c: $N \cdot d \ge b_N(c) \ge b_{N-1}(c) \ge \cdots \ge b_1(c) = d$.

Graphical representation of optimal transmission policy



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<u>Question</u>:

Can we find explicit form for critical numbers?

Single User Case
Complete Characterization of Optimal Policy

Additional Technical Assumptions

- Set of possible channel conditions is finite: $\mathcal{C} = \{c_1, c_2, \ldots, c_K\}$
- Receiver buffer empty at beginning of time horizon
- $L(c) := P/(c \cdot d)$ is an integer

Thresholds & Critical Numbers

- We can define recursively a set of thresholds
- From these thresholds, we can find the critical numbers
- This process is far simpler computationally than solving the dynamic program

Single User Case
Infinite Horizon Problem

- Infinite horizon optimal policy is natural extension of finite horizon optimal policy

- Stationary optimal policy characterized by critical numbers $b_\infty(c)$, where

$$b_\infty(c) := \lim_{n \to \infty} b_n(c)$$

- Again, we have a modified base-stock policy:

$$y_\infty^*(x,c) := \begin{cases} x\,, & if\;\; x \geq b_\infty(c) \\ b_\infty(c)\,, & if\;\; b_\infty(c) - \dfrac{P}{c} \leq x \leq b_\infty(c) \\ x + \dfrac{P}{c}\,, & if\;\; x < b_\infty(c) - \dfrac{P}{c} \end{cases} .$$

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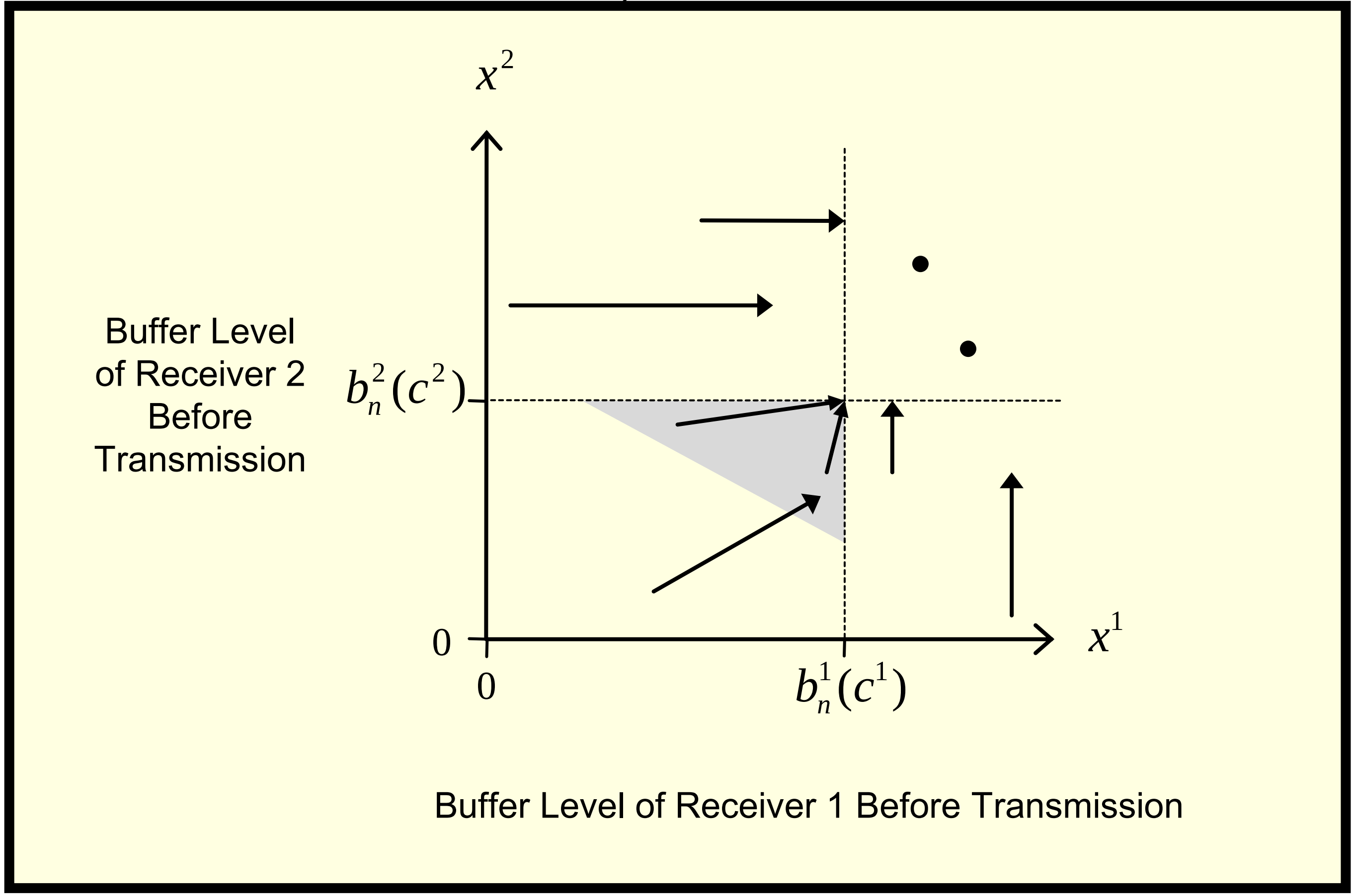
Multiple Users Case
Conjecture Based on Numerical Experiments

Single user optimal policy appears to extend in following manner:

- For each vector of channel conditions ***c*** at time *n*, there exists a vector of critical numbers with one critical number for each user
  - Each user's critical number b_n^m depends only on its current channel condition c^m
    - Independent of its own current buffer level
    - Independent of other users' current buffer levels
    - Independent of other users' current channel conditions
- Optimal policy is characterized by the critical numbers
  - Do not transmit packets to any user whose current buffer level exceeds its critical number
  - If possible to bring all users up to critical number, do so
  - If power constraint prevents transmitter from bringing all users up to their critical numbers, it should allocate the full power *P* to different users
    - Yet to determine optimal allocation between users

Multiple Users Case Example

Structure of the optimal policy for the two-user case in slot n, with a fixed vector of channel conditions, c.



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Current and Future Work

- Prove structure of optimal policy in multiple users case
- Identify low-complexity sub-optimal policies that perform well in multi-user case when power constraint prevents transmitting up to all critical numbers
- Performance analysis
- Derive alternate condition for complete characterization of optimal policy in single user case
  - Set the maximum number of packets that can be transmitted in a slot to $\left\lfloor \frac{P}{c \cdot d} \right\rfloor \cdot d$
- Relax *i.i.d.* assumption, and examine Markovian channel
- Explore the possibility of dropping resource-draining users and determining when new users can be admitted

Concluding Remarks

- Considered problem of transmitting media streams over a shared wireless channel in a manner that prevents receivers' buffers from emptying
- In single user case, showed optimal transmission schedule is a modified base-stock policy under both finite and infinite horizon discounted expected cost criteria
- Numerical experiments suggest similar structure for multi-user case
- Modified base-stock policies have nice feature that they are easily implementable