Problem 1: A rectangular sheet has charge $Q$ and sides $2a$ and $2b$.

(A) Find the magnitude and direction of the electric field a distance $x_0$ perpendicular from the center of the sheet along the $x$ axis. Hint:

$$
\int \frac{1}{(k^2 + u^2)^{3/2}} \, du = \frac{1}{k^2} \frac{u}{\sqrt{k^2 + u^2}}
$$

$$
\int \frac{1}{(u^2 + v^2)\sqrt{u^2 + v^2 + k^2}} \, dv = \frac{1}{ku} \tan^{-1} \frac{kv}{u\sqrt{u^2 + v^2 + k^2}}
$$
(B) Show that when \( x_0 << a \) and \( x_0 << b \), the expression reduces to that of a charge distribution on a plane. Make sure the approximation makes sense to you.

(C) Show that when \( x_0 >> a \) and \( x_0 << b \), the expression reduces to that of a charge distribution on a line. Make sure the approximation makes sense to you.

(D) Show that when \( x_0 >> a \) and \( x_0 >> b \), the expression reduces to that of a point charge. Make sure the approximation makes sense to you.

(E) Plot the complete expression you found in (A) using a log-log scale to see the dependence on distance. Identify the three regions derived in the previous three parts. \textit{Hint: You must assume } a >> b \textit{ and make sure } x_0 \textit{ crosses all regions.}

(F) Show that when \( b \to 0 \), the expression reduces to that of a finite line charge distribution, as derived in class.

(G) Find the resulting expression when \( a = b \). What should the expression reduce to in the limits \( x_0 >> a \) and \( x_0 << a \)? Show that it behaves as expected.