Problem 1:

A. Find an expression for $F[f_v(t)]$, where

\[ v(t) = \begin{cases} V & -T_1 + nT < t < T_1 + nT, n \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases} \]

*Hint: In lecture 6, we showed that convolution with a shifted impulse creates a copy of the original signal shifted by that amount. Use this to express $v(t)$ as the convolution of a pulse and an impulse train, as shown below.*

\[ v(t) = \left\{ \begin{array}{ll} V & -T_1 < t < T_1 \\ 0 & \text{otherwise} \end{array} \right\} \ast \sum_{k=-\infty}^{+\infty} \delta(t - kT) \]

B. Graph $F\{v(t)\}$ for the case $T_1 = T/4$ and $V = V_S$.

C. Show that the Fourier transform you found is equivalent to the coefficients for the even square wave with period $T$ and pulse width $2T_1$,

\[ c_n = 2V \frac{T_1}{T} \text{sinc} \left( 2\pi n \frac{T_1}{T} \right) \]

such that

\[ v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j \frac{2\pi}{T} n t}. \]
D. Optional: In Quiz 5, you found that the frequency content of a period $T$ function only exists in the harmonics of $2\pi/T$ (multiples of $\frac{2\pi}{T}$) and must be zero elsewhere. Can you generalize the transform of a periodic function $x(t) = x(t + T)$? For consistency of notation, define $x_T(t)$ as a single period of $x(t)$ and $X_T(j\omega) = \mathcal{F}\{x_T(t)\}$.

E. Optional: Generalize the equivalence to series

$$c_n = \frac{1}{T} \cdot X_T \left( j\frac{2\pi}{T}n \right).$$