**Problem 1:** Show Parseval’s Theorem

\[
\int_{t=-\infty}^{t=\infty} |x(t)|^2 dt = \int_{\omega=-\infty}^{\omega=\infty} |X(j\omega)|^2 \frac{d\omega}{2\pi}
\]

where

\[|X(j\omega)|^2 = X(j\omega)X^*(j\omega)\]

\(x(t)\) is real, and \(X^*(j\omega)\) is the complex conjugate of \(X(j\omega)\).

*Hint:*

\[X^*(j\omega) = \int_{t=-\infty}^{t=\infty} x(t)e^{-j\omega t} dt\]

**Problem 2:** A non-ideal filter or communication channel \(H(j\omega)\) with finite *transition band* \(\Delta f\) between the *passband* (where \(H(j\omega) \neq 0\)) and *stopband* (where \(H(j\omega) = 0\)) can be modeled as \(H(j\omega) = H_0(j\omega) \ast H_{\Delta}(j\omega)\) where

\[
H_0(j\omega) = \begin{cases} 
1 & -2\pi f_0 < \omega < 2\pi f_0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
H_{\Delta}(j\omega) = \begin{cases} 
1 & -2\pi \Delta f/2 < \omega < 2\pi \Delta f/2 \\
0 & \text{otherwise}
\end{cases}
\]

A. Find an expression for \(h_0(t)\), the impulse response of the ideal channel.

B. Find an expression for \(H(j\omega)\) and sketch it.

C. Find an expression for the impulse response \(h(t)\) of the non-ideal channel.

D. Plot \(h(t)\) and \(h_0(t)\) on the same axes when \(f_0 = 10\) kHz and \(\Delta f = 500\) Hz and sketch the associated \(H(j\omega)\). Repeat the plot for the same values when \(\Delta f = 100\) Hz. Compare the plots. For what values of \(\Delta f\) is the effect of the non-ideal transition band noticeable? Is the effect what you would expect? Explain.
Problem 3: The system shown below represents a basic communication system where two messages $x_1(t)$ and $x_2(t)$ share a common communication channel. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to $f_B$ and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of $f_B$ as shown below.

A. What would happen if $\omega_1 = \omega_2 = 0$? Find $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$, and show its frequency content.

B. Find constraints on $\omega_1$ and $\omega_2$ such that there is no frequency interference (aliasing). Show the frequency content of $m(t)$ and $d(t)$ under these constraints. Note: There may be multiple solutions; just find one that works.

C. Show the frequency content and find an algebraic expression for $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$ assuming the constraints of part B.