Franklin W. Olin College of Engineering
Needham, Massachusetts
Electrical and Computer Engineering

ENGR2410 – Signals and Systems

Quiz 3

Instructions

A. Collaboration is not allowed on quizzes.
B. Students may only use a page of notes during the quizzes.
C. Time is limited to one continuous hour.
D. Quizzes are due at the end of lecture on Thursday.
E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.
F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.
G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.
Problem 1:

A. Find the transfer function for the circuit shown below using impedances.

Solution:

\[
H(s) = \frac{V_O}{V_I} = \frac{Ls + \frac{1}{C_s}}{R + Ls + \frac{1}{C_s}}
\]

\[
H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{1}{L/R}s + \frac{1}{LC}}
\]

\[
\omega_0^2 = \frac{1}{LC} \quad \alpha = \frac{1}{2L/R}
\]

\[
H(j\omega) = \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2 + j2\alpha\omega}
\]
B. Sketch only the Bode plot of the magnitude of $H(s)$.

Solution:

$\omega \to 0 \quad H(j\omega) \approx 1$

$\omega \to \infty \quad H(j\omega) \approx 1$

At resonance, $\omega = \omega_0$, and the series LC acts like a short.

$\omega = \omega_0 \quad H(j\omega) = 0$

Only the magnitude is required for the quiz.
C. Recall that at low frequencies a capacitor may be replaced with an open circuit and an inductor may be replaced with a short circuit and the inverse is true at high frequencies. Draw equivalent circuits for low frequencies and high frequencies. Use them to verify the extremes of the Bode plot.

**Solution:**
The left circuit depicts how the original circuit behaves at low frequencies when the capacitor acts like an open circuit and the inductor acts like a short. The right circuit depicts the original circuit at high frequencies when the capacitor acts like a short and the inductor acts like an open circuit. The result in either of these cases is that $v_O = v_I$ as expected, since current flows through R.
D. Draw an equivalent circuit at resonance.

**Solution:**

\[ v_O = 0, \text{ or } H(s) = 0. \]

At resonance, \( v_O = 0 \), or \( H(s) = 0 \).
**Problem 2:** SUPER optional problem that will *not* be graded, so do it for fun if you want to! What is the equivalent impedance of an infinite chain of series LC? How does it Bode plot look like? Can it be interpreted physically?

**Partial solution:**

\[ Z = L_s + \frac{1}{C_s} || Z \]

\[ Z = L_s + \frac{Z}{ZC_s + 1} \]

\[ Z^2 - L_s Z - L/C = 0 \]

\[ \alpha = -L_s/2 \quad \omega_0 = -L/C \]

\[ Z = L_s/2 \pm \sqrt{(Ls/2)^2 + L/C} \]
Name (optional):

**Problem 3:** Since we are trying a new format for the course, we need your help to assess its impact. Feel free to send any additional feedback directly to us. Please make sure this page is printed by itself so that we may keep it when we return the graded quiz to you.

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?