Instructions

A. Collaboration is not allowed on quizzes.
B. Students may only use a page of notes during the quizzes.
C. Time is limited to one continuous hour.
D. Quizzes are due at the end of lecture on Thursday.
E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.
F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.
G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.
Problem 1: For consistency throughout this problem, sketch the all Fourier transforms from -30 kHz to 30 kHz.

A. Sketch the Fourier transform of \( x(t) = 2\cos(2\pi \cdot 1 \text{ kHz} \cdot t) + \cos(2\pi \cdot 8 \text{ kHz} \cdot t) \).

B. \( x(t) \) is sampled at 20 kHz. Sketch the Fourier transform of the resulting function, \( x_{s1}(t) \).

C. \( x_{s1}(t) \) is passed through an ideal low-pass filter from -10 kHz to 10 kHz. Write an expression for the filtered output \( y_1(t) \).
D. $x(t)$ is sampled at 10 kHz. Sketch the Fourier transform of the resulting function, $x_{S2}(t)$.

E. $x_{S2}(t)$ is passed through an ideal low-pass filter from -10 kHz to 10 kHz. Write an expression for the filtered output $y_2(t)$.

A fascinating debate is whether the human brain samples what we see, and if so, what is the sampling frequency. Check out all the sampling language in this letter. However, newer evidence like this article suggests that sampling may not be enough to explain our perception.
Problem 2: Find an algebraic expression for the inverse Fourier transform of $X(j\omega)$. This filter is known as a raised-cosine filter and is very important in digital communications. For example, check out this article. You just found the impulse response of the filter.

$$X(j\omega) = \begin{cases} \frac{1}{2} \left[ 1 + \cos \left( \frac{\omega}{2f_S} \right) \right] & -2\pi f_s \leq t \leq 2\pi f_s \\ 0 & \text{otherwise} \end{cases}$$
Fourier Transform

If \( x(t) = \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \frac{d\omega}{2\pi} \) then \( X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \triangleq \mathcal{F}\{x(t)\} \)

\[
\begin{align*}
\mathcal{F}\{ax_1(t) + bx_2(t)\} &= aX_1(j\omega) + bX_2(j\omega) \\
\mathcal{F}\{x(t + T)\} &= X(j\omega) e^{j\omega T} \\
\mathcal{F}\{\delta(t)\} &= 1 \\
\mathcal{F}\{1\} &= 2\pi\delta(\omega) \\
\mathcal{F}\{e^{j\omega_0 t}\} &= 2\pi\delta(\omega - \omega_0) \\
\mathcal{F}\{\cos(\omega_0 t)\} &= \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0) \\
\mathcal{F}\{\sin(\omega_0 t)\} &= \frac{\pi}{j}\delta(\omega - \omega_0) - \frac{\pi}{j}\delta(\omega + \omega_0) \\
\mathcal{F}\left\{\frac{1}{2}\delta(t - T) + \frac{1}{2}\delta(t + T)\right\} &= \cos(\omega T) \\
\mathcal{F}\{e^{-t/\tau}u(t)\} &= \frac{1}{\tau + j\omega} \\
\mathcal{F}\{x(t) \ast y(t)\} &= X(j\omega)Y(j\omega) \\
\mathcal{F}\{x(t)y(t)\} &= \frac{1}{2\pi} X(j\omega) \ast Y(j\omega) \\
\mathcal{F}\{\Pi(t/T_1)\} &= 2T_1 \frac{\sin(\omega T_1)}{\omega T_1} \text{ where } \Pi(x) = \begin{cases} 
1 & -1 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases} \\
\mathcal{F}\left\{\frac{\omega_0}{\pi} \cdot \frac{\sin(\omega_0 t)}{\omega_0 t}\right\} &= \Pi(\omega/\omega_0) \text{ where } \Pi(x) = \begin{cases} 
1 & -1 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases} \\
\mathcal{F}\left\{\sum_{n=-\infty}^{\infty} \delta(t - nT)\right\} &= \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta \left(\omega - k\frac{2\pi}{T}\right) \text{ where } n \text{ and } k \text{ are integers}
\end{align*}
\]
Problem 3: Since we are trying a new format for the course, we need your help to assess its impact. Feel free to send any additional feedback directly to us. Please make sure this page is printed by itself so that we may keep it when we return the graded quiz to you.

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?