Instructions

A. Collaboration is not allowed on quizzes.

B. Students may only use a page of notes during the quizzes.

C. Time is limited to one continuous hour.

D. Quizzes are due at the end of lecture on Thursday.

E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.

F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.

G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.
Problem 1: The system shown below introduces the concept of quadrature, where we send multiple signals that share not only a common channel, but also the same frequency band. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to $f_B$ and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of $f_B$ as shown below.

\[ H(j\omega) \]

\[ X_1(j\omega) \]

\[ X_2(j\omega) \]

in-phase (I) channel

\[ x_1(t) \]

\[ x_2(t) \]

\[ \cos(\omega t) \]

\[ \sin(\omega t) \]

quadrature (Q) channel
A. Show that the frequency content of \( M(j\omega) \) is

\[
M(j\omega) = \frac{1}{2}X_1(\omega - \omega_1) + \frac{1}{2}X_1(\omega + \omega_1) - j\frac{1}{2}X_2(\omega - \omega_1) + j\frac{1}{2}X_2(\omega + \omega_1)
\]

The result can be either the expression above or neatly labeled sketches of both the real and imaginary parts of \( M(j\omega) \). *Hint:* You can use either equations or sketches to find the solution, but using both may help you avoid algebra mistakes.
B. Find an expression for $y(t)$. Justify your answer clearly.
C. *Bonus:* Find an expression for $y(t)$ if $m(t)$ is multiplied by $\sin(\omega_1 t)$ in the receiver instead of multiplied by $\cos(\omega_1 t)$. *Hint:* You can use your intuition (and should!) to guess the answer, but a clear justification will get more points.
Continuous time Fourier transform

If \( x(t) = \int_{\omega=-\infty}^{\omega=\infty} X(j\omega)e^{j\omega t} \frac{d\omega}{2\pi} \) then \( X(j\omega) = \int_{t=-\infty}^{t=\infty} x(t)e^{-j\omega t} dt \triangleq \mathcal{F}\{x(t)\} \)

Alternatively, \( x(t) \overset{\mathcal{F}}{\leftrightarrow} X(j\omega) \)

System representation

\[
x(t) \quad \overset{\mathcal{F}}{\longrightarrow} \quad h(t) \quad \overset{\mathcal{F}}{\longrightarrow} \quad y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\tau=\infty} x(\tau)h(t-\tau)d\tau
\]

\[
X(j\omega) \quad \overset{\mathcal{F}}{\longrightarrow} \quad H(j\omega) \quad \overset{\mathcal{F}^{-1}}{\longrightarrow} \quad Y(j\omega) = X(j\omega)H(j\omega)
\]

Properties and transforms

\[
\mathcal{F}\{ax_1(t) + bx_2(t)\} = aX_1(j\omega) + bX_2(j\omega)
\]

\[
\mathcal{F}\{x(t + T)\} = X(j\omega)e^{j\omega T}
\]

\[
\mathcal{F}\{\delta(t)\} = 1
\]

\[
\mathcal{F}\{1\} = 2\pi\delta(\omega)
\]

\[
\mathcal{F}\{e^{j\omega_0 t}\} = 2\pi\delta(\omega - \omega_0)
\]

\[
\mathcal{F}\{\cos(\omega_0 t)\} = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)
\]

\[
\mathcal{F}\{\sin(\omega_0 t)\} = \frac{\pi}{j}\delta(\omega - \omega_0) - \frac{\pi}{j}\delta(\omega + \omega_0)
\]

\[
\mathcal{F}\left\{\frac{1}{2}\delta(t - T) + \frac{1}{2}\delta(t + T)\right\} = \cos(\omega T)
\]

\[
\mathcal{F}\{e^{-t/\tau}u(t)\} = \frac{1}{\tau + j\omega}
\]

\[
\mathcal{F}\{x(t) * y(t)\} = X(j\omega)Y(j\omega)
\]

\[
\mathcal{F}\{x(t)y(t)\} = \frac{1}{2\pi} X(j\omega) * Y(j\omega)
\]

\[
\mathcal{F}\{\Pi(t/T_1)\} = 2T_1 \sin(\omega T_1) \frac{\omega T_1}{\omega_0 T_1} \text{ where } \Pi(x) = \begin{cases} 1 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
\mathcal{F}\left\{\frac{\omega_0}{\pi} \cdot \frac{\sin(\omega_0 t)}{\omega_0 t}\right\} = \Pi(\omega/\omega_0) \text{ where } \Pi(x) = \begin{cases} 1 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
\mathcal{F}\left\{\sum_{n=-\infty}^{\infty} \delta(t - nT)\right\} = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta\left(\omega - k\frac{2\pi}{T}\right) \text{ where } n \text{ and } k \text{ are integers}
\]
Discrete time Fourier transform \((n \in \mathbb{Z})\)

If \(x[n] = \int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega\) then \(X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}\)

Alternatively, \(x[n] \xrightarrow{\mathcal{F}} X(\Omega)\)

System representation

\[
\begin{align*}
x[n] & \quad \xrightarrow{\mathcal{F}} \quad h[n] \quad \xrightarrow{\mathcal{F}} \quad y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
X(\Omega) & \quad \xrightarrow{\mathcal{F}} \quad H(\Omega) \quad \xrightarrow{\mathcal{F}} \quad Y(\Omega) = X(\Omega)H(\Omega)
\end{align*}
\]

Properties and transforms

\[
\delta[n] = \begin{cases} 
1 & \text{if } n = 0 \\
0 & \text{otherwise} 
\end{cases} \quad \xrightarrow{\mathcal{F}} \quad 1
\]

\[
x[n-n_0] \quad \xrightarrow{\mathcal{F}} \quad X(\Omega)e^{-j\Omega n_0}
\]

\[
e^{j\Omega_0 n} \quad \xrightarrow{\mathcal{F}} \quad \sum_{k} 2\pi \delta(\Omega - \Omega_0 - 2\pi k), \quad k \in \mathbb{Z}
\]

\[
a^n u[n] \quad \xrightarrow{\mathcal{F}} \quad \frac{1}{1-ae^{-j\Omega}}, \quad |a| < 1
\]

DT processing of CT signals

\[
x_c(t) \quad \xrightarrow{C/D} \quad x_d[n]=x_c(n/fs) \quad \xrightarrow{H_d(\Omega)} \quad y_d[n]=y_c(n/fs) \quad \xrightarrow{C/D} \quad y_c(t)
\]

Sampling at \(f_s\) constrains \(\Omega\) such that \(\frac{\omega}{f_s} = \Omega\). If \(x_c(t)\) is bandlimited by \(f_{max}\) such that the sampling frequency \(f_s > 2f_{max}\), the system above is equivalent to an LTI system \(H_c(j\omega)\), where

\[
H_c(j\omega) = \begin{cases} 
H_d(\omega/f_s) & -2\pi \frac{f_s}{2} \leq \omega \leq 2\pi \frac{f_s}{2} \\
0 & \text{otherwise}
\end{cases}
\]

Equivalently, \(Y_c(j\omega) = H_d(\omega/f_s)X_c(j\omega)\)
Name (optional):

**Problem 2:** Since we are trying a new format for the course, we need your help to assess its impact. Feel free to send any additional feedback directly to us. Please make sure this page is printed by itself so that we may keep it when we return the graded quiz to you.

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?