**Problem Set 4**  
Issued 23 Feb 2015  
**Homework**  
Due 26 Feb 2015

**Problem 1:** For the following loop transfer functions in unity feedback, use approximate asymptotic methods to find the magnitude of the closed-loop frequency response by hand.

\[
L_a(s) = \frac{0.25s}{(s+1)(0.01s+1)^2}
\]

\[
L_b(s) = \frac{25s}{(s+1)(0.01s+1)^2}
\]

\[
L_c(s) = \frac{\pi(s+1)}{s^2(0.1s+1)(0.01s+1)}
\]

**Problem 2:** Construct asymptotic Bode plots (magnitude and phase) by hand for the following loop transfer functions and show the location of the applicable stability margins (gain margin, phase margin, \(\omega_{\phi}\), and \(\omega_c\)). Construct a table listing the stability margins.

\[
L_d(s) = \frac{20}{0.2s+1}
\]

\[
L_e(s) = \frac{20}{(0.2s+1)(0.1s+1)}
\]

\[
L_f(s) = \frac{100}{s(0.1s+1)(0.01s+1)}
\]

**Problem 3:** Use MATLAB to verify your results from Problem 2.

**Problem 4:** on next page
**Problem 4:** You are given the following set of performance specifications for a unity-gain feedback system

- Crossover frequency of $\omega_c > 10^4$ rps with at least $45^\circ$ of phase margin.
- Steady state error for a ramp input less than or equal to 1%.
- For sinusoidal inputs over the range $0 < \omega < 10^3$ rps.

\[ \left| \frac{E(j\omega)}{R(j\omega)} \right| < 1.5\% \]

- For sensor noise variations in the frequency range $\omega > 10^5$ rps,

\[ \left| \frac{C(j\omega)}{N(j\omega)} \right| < 7\% \]

- For sensor noise variations in the frequency range $\omega > 10^6$ rps,

\[ \left| \frac{C(j\omega)}{N(j\omega)} \right| < 0.1\% \]

(a) Draw the Bode obstacle course for these specifications.

(b) Construct an asymptotic Bode magnitude curve which will meet these specifications with as little extra magnitude as possible. Give an expression for a rational function $L(s)$ which will exhibit the same asymptotic curve that you have just constructed.

(c) Use MATLAB to verify your control-system performance.