Problem 1: Sketch root-locus plots for the following systems. Find the departure angles, asymptote angles, and centroids.

\[
\begin{align*}
L_a(s) &= \frac{K}{(s+1)^3} & L_b(s) &= \frac{Ks}{(s+1)^3} & L_c(s) &= \frac{K}{s(s+1)^3} \\
L_d(s) &= \frac{K}{(s+1)^4} & L_e(s) &= \frac{Ks}{(s+1)^4} & L_f(s) &= \frac{K}{s(s+1)^4}
\end{align*}
\]

Problem 2: In lecture, we discussed the compensation of a plant transfer function

\[ G_p(s) = \frac{K}{s} \left( \frac{10^6}{s^2 + 20s + 10^6} \right) \]

with a compensator transfer function that adds \( N \) more poles

\[ G_c(s) = \frac{1}{(\tau s + 1)^N} \]

such that the departure angles from the complex-conjugate poles is \( \delta = -180^\circ \). In lecture, we found that for \( N = 4 \), \( \tau = 0.001 \) s does the trick.

\[ L(s) = G_c(s)G_p(s) \]

(a) Find \( \tau \) for \( N = 3 \), \( N = 5 \), and \( N = 6 \), such that the departure angles are maintained at \( \delta = -180^\circ \).

(b) Use Matlab to plot the root locus for \( N = 3 \), \( N = 4 \), \( N = 5 \), and \( N = 6 \) (with the respective time constants) to check your results.

(c) Plot the closed-loop pole-zero plots and closed-loop step responses for all four systems, with \( K = 200 \). Comment on the trend (or lack thereof) in the step responses.

\[ \frac{C}{R}(s) = \frac{L(s)}{1 + L(s)} \]
Problem 3: This problem will explore some of the nuances of root locus in systems with non-unity feedback.

(a) As a matter of familiarizing you with the system shown above, set $G_c(s) = 1$ and $H(s) = 1$. Plot the root locus of the loop transfer function $L(s)$, and specify the range of positive $K$ for which the system is stable.

(b) Since this system is stable for an unacceptably small range of $K$, your friend Edward suggests one method of compensation; you follow his suggestion and set $G_c(s) = (s + 4)$ while leaving $H(s) = 1$. Plot the root locus of $L(s)$ with this compensator in place, and plot the locations of the closed-loop poles and zeros for $K = 200$. Additionally, use the `step` command in Matlab to plot the step response of the closed-loop system with $K$ still fixed at 200.

(c) As you are boasting about Ed’s clever method of compensation, your other friend Harold mentions that he has thought of another, more attractive, way of compensating the system. He suggests setting $G_c(s) = 1$, but making $H(s) = (s + 4)$. You are intrigued, and decide to compare Harold’s scheme to Ed’s configuration. Plot the root locus of $L(s)$ with this compensator in place, and plot the locations of the closed-loop poles and zeros for $K = 200$. Plot the step response of this system in Matlab as well with the same value of $K$ as you were previously using.

(d) Are the step responses different? If so, provide a short explanation for why this might be so.