Exercise 3: Similarity and Dedimensionalization

Transport in Biological Systems

Fall 2015

These problems are due by Thursday, October 29th in class. Please feel free to discuss them with your colleagues, but make sure your work represents your own understanding of how to apply the concepts we've been discussing in class.

1. A bioengineer wants to study the flow of blood in a lifelike (i.e., full scale) model of the aortic bifurcation (where the aorta splits to form the vessels that perfuse the legs), but must use a working fluid whose kinematic viscosity, ν, is 6 centistokes; the kinematic viscosity of blood is about 4 cs. The densities of the working fluid and blood are both approximately 1 g/ml. The engineer wants to simulate a mean blood flow of 15 ml/sec and a pulse rate of 65/min.
   (a) What should the mean flow rate and pulse rate be in the model?
   (b) At a particular site in the model, the measured shear stress is 20 dynes/cm^2. What would the shear stress in the artery be at the same site?
2. An aspiring graduate student wants to study the flow of blood in the coronary arteries, but because a typical vessel is only 3 mm in diameter and too small to make measurements in, she makes a replica that is four times the size of real coronary arteries. Based on published data for coronary artery dimensions and flow, and the known properties of blood, she calculates that the time-average Reynolds number of the in vivo flow is 90 and the Womersley number (look this up in the book) is 3.1. The pump she is using to drive flow system has a fixed period of 3 sec.
   (a) What should be the kinematic viscosity of her working fluid?
   (b) Based on the values given here, what is the average in vivo inlet velocity? What should the average velocity be at the inlet to the model to obtain similarity to the in vivo case?
3. Consider a combination of steady plane Couette flow (i.e. flow caused by one plate moving relative to the other at velocity U) in the presence of a constant pressure gradient (which by itself is Poiseuille flow). Once fully developed, the fluid velocity vector is parallel to the bounding plates, but varies with location (y) between the plates. The fluid is Newtonian. As in previous examples, assume the plate has some width, w.
   (a) Write the full x-momentum Navier-Stokes equation and indicate which terms are zero for this problem, and why.
   (b) Solve for the velocity profile between the plates and the shear rates at the two plates.
   (c) What would the relationship between U and P have to be for the average velocity to be zero?
4. Write down the full Navier-Stokes equation or one of the components in a coordinate system of your choice. Define dimensionless variables (e.g. r^*) and de-dimensionalize the equation. (There is a typo in my version of the book. If you have that version, what is it?)

Note: Problems 1-3 are modified from Profs Holmes (Columbia) and/or Moore (Texas A&M).