# Exercise 4: More problems 

Transport in Biological Systems

Fall 2015

These problems are due by Thursday, November 19th. Please feel free to discuss them with your colleagues, but make sure your work represents your own understanding of how to apply the concepts we've been discussing in class.

1. One approach to tissue engineering blood vessels is to wrap sheets of cells around a central mantril tube. Because vessels are composed of different tissue layers, tissue engineers often create an inner muscular layer (the media) and an outer connective tissue layer (the adventitia). One of the oftcited limitations of engineering is the limitation of diffusion of nutrients and oxygen. Two potential solutions are to vascularize (grow capillaries into) the tissue or to make it thin enough that the cells in the center can receive nutrients. You are a considering which approach to take and have control over both layers.
(a) Towards designing your blood vessel, consider a two-layer model of a blood vessel with an inner radius, $R_{i}$, an outer radius, $R_{o}$. Also let $R_{1}$ be the outer radius of the inner layer (and the inner radius of the outer layer).
(b) The inner and outer layers have a diffusion coefficients $D_{1}$ and $D_{2}$, respectively.
(c) The concentrations of the relevant solutes in and outside the vessel are $C_{i}$ and $C_{o}$, respectively.
(d) What is the effective diffusion coefficent?
(e) Start by writing expressions for the concentrations in the inner and outer vessel layers, $C_{1}$ and $C_{2}$, respectively.
(f) Get your effective diffusivity into a form so that you could figure out an effective diffusivity for additional layers. Note, the solution should reduce to the expression for a single layer vessel if the diffusivities are the same. (Revised version of problem 6.10 from the book.)
2. Work through example 6.8 in the book and then do Problem 6.14 from the book.
3. Problem 9.6 from Truskey.
4. Compare the fluid mechanic nature of flow in the human aorta versus that in the rabbit aorta based on the following information. Assume a blood viscosity of $\nu=0.04 \mathrm{~cm}^{2} / \mathrm{sec}$ in both cases. (Modified from Professor Moore at Texas A\&M.)

|  | Human | Rabbit |
| :--- | :---: | :---: |
| Cardiac output | $5 \mathrm{l} / \mathrm{min}$ | $300 \mathrm{ml} / \mathrm{min}$ |
| Heart rate | 72 bpm | 200 bpm |
| Aortic diameter | 2.5 cm | 3 mm |

5. Pressure drop vs. flow rate data have been collected in the straight tube upstream section of an aorta flow model. The straight tube diameter is 2 cm , and the pressure measurements were done at axial positions 5 cm apart. Water is the working fluid (kinematic viscosity $=0.01 \mathrm{~cm}^{2} / \mathrm{s}$ ). Can you fit a
straight line through this data? Should you be able to? If a straight line fit does not work, explain why. (Modified from Professor Moore at Texas A\&M.)

| $\Delta P\left(\mathrm{~g} / \mathrm{cm}^{2} \mathrm{~s}^{2}\right)$ | $Q\left(\mathrm{~cm}^{3} / \mathrm{s}\right)$ |
| :---: | :---: |
| 1.5 | 12 |
| 3.8 | 30 |
| 12 | 55 |

Note: Problems 1-3 are modified from Professor Holmes at Columbia and/or Professor Moore at Texas A\&M and modified slightly for our course.

