Exercise 5: More problems

Transport in Biological Systems

Fall 2015

These problems are due by Thursday, December 10th. Please feel free to discuss them with your colleagues, but make sure your work represents your own understanding of how to apply the concepts we've been discussing in class.

1. Use the equation for the velocity profile in tube flow to express the shear stress at the wall in terms of the pressure gradient $\Delta P/L$, the viscosity of the fluid and the radius of the tube.
   (a) Then use this answer with the Poiseuille equation ($Q = \Delta P \pi R^4/8\mu L$) to express the shear stress in terms of the flow rate and the vessel radius. Note that the direction away from the wall and into the flow, analogous to y in the 2-D case, is -r.
   (b) A typical mean blood velocity in the abdominal aorta is 15 cm/sec. The vessel is typically 1 cm in diameter. As we have discussed, the blood viscosity is approximately 4 centipoise (cp). What is the shear stress at the wall of the abdominal aorta in $dynes/cm^2$?
2. In class, we took a differential mass balance approach to develop an expression for the velocity profile for our favorite system – two parallel plates two parallel plates that are much longer and wider (x and z directions) than the gap between them, h in the y direction. – with one moving at speed, U. Now take a more formal approach and develop the expression using Navier-Stokes. Assume that the fluid is Newtonian and incompressible, flow is steady and laminar, and $\partial P/\partial x = 0$. Given this:
   (a) Solve for the nonzero components of v and τ.
   (b) Graph these components.
   (c) Derive an expression for the volumetric flow rate, Q, in terms of upper plate speed, U, channel height, h, and assumed channel width, w.
   (d) Find the force required to keep the upper plate moving at a constant speed.
   (e) Why is this a good setup to measure viscosity?
3. Now, for our same parallel plate system and assumptions about the fluid, write and simplify the Navier-Stokes equations if fluid movement is driven by an oscillating pressure gradient in the x direction of flow.
   (a) What can you conclude by inspection? What assumptions do we need to make?
   (b) Write out the appropriate components of the Navier-Stokes equations and simplify them appropriately, explaining which terms you can eliminate and which ones you keep.
   (c) Specify the boundary conditions.
   (d) You can stop here or, for extra credit, solve the resulting differential equation.
4. You are studying muscle physiology and need to design a set up make sure no part of the muscles you are studying becomes too hypoxic to contract ($P_{O2} < 20$ mm Hg). You wish to place thin, cylindrical

muscles in a bath containing oxygenated, stirred saline. You can achieve an oxygen partial pressure of 600 mmHg by bubbling oxygen into the bath. You look up values in the literature for oxygen solubility (0.024 ml O_2 / ml tissue.atm) and diffusion coefficient (1.0 - 3.0 x 10^{-5} cm^2/s) in muscle. You expect muscle to consume 2.0 - 10.0 ml O_2 / 100 g tissue.min.

(a) Solve the diffusion equations for this problem and find an expression for the maximum allowable muscle radius, R.

(b) Given the constants you found in the literature, what is the allowable radius for muscles to be studied in your bath? Does this seem reasonable?

Problems modified from Professors Holmes (Columbia) or Moore (TAMU).