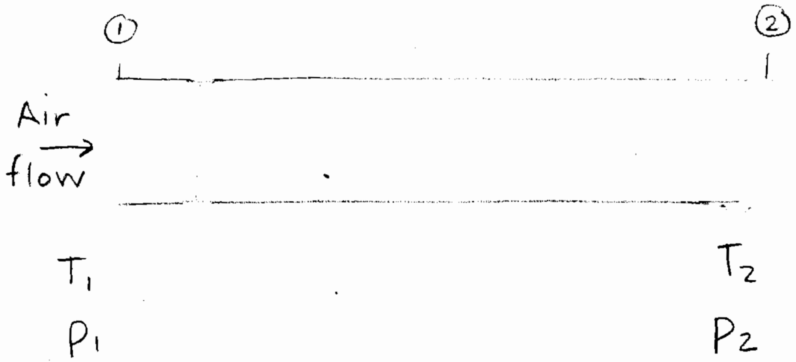
Lecture 19

Oct. 14, 1947
Bell X-1
Chuck Yeager flew sonic for the first time

Compressible flow (con'd)

Pg 75 from notes first - review of thermo

Example:



Constant specific heats
C_v, C_p

a) Δu (internal energy)

$$u_2 - u_1 = c_v (T_2 - T_1)$$

b) Δh (enthalpy)

$$h_2 - h_1 = c_p (T_2 - T_1)$$

c) $\Delta \rho$ (density)

$$\rho_2 - \rho_1 = \frac{P_2}{RT_2} - \frac{P_1}{RT_1}$$

d) entropy change

$$s_2 - s_1 = \underbrace{c_p \ln \frac{T_2}{T_1}}_{-} - \underbrace{R \ln \frac{P_2}{P_1}}_{-}$$

$T_2 < T_1$
$P_2 < P_1$

Reminder: frictionless, adiabatic flow of any fluid $\rightarrow \Delta s = 0$

$$c_p \ln \frac{T_2}{T_1} = R \ln \frac{P_2}{P_1}$$

$$\left(\frac{T_2}{T_1}\right)^{c_p} = \left(\frac{P_2}{P_1}\right)^R$$

$$\frac{c_p}{R} = \frac{k}{k-1}$$

$$\left(\frac{T_2}{T_1}\right)^{k/k-1} = \frac{P_2}{P_1}$$

$$\frac{P}{\rho^k} = \text{constant}$$

> Speed of sound, c

- Rate of propagation of a pressure pulse of infinitesimal strength through a still fluid (thick of pressure wave in a gas $\approx 10^{-6}$ ft)

$$c = \sqrt{\frac{\partial P}{\partial \rho}}\Bigg|_s$$

$$\frac{\partial P}{\partial \rho} = (\text{constant})\, k\rho^{k-1} = \frac{P}{\rho^k} k\rho^{k-1} = \frac{P}{\rho} k$$

$$= kRT$$

$$c = \sqrt{kRT}$$

Compressible Isentropic Flow of an Ideal Gas (adiabatic, frictionless)

Stagnation Pressure $\quad P_o = P + \frac{1}{2}\rho V^2$

Stagnation Temperature $\quad T_o = T + \frac{V^2}{2c_p}$

temperature + pressure achieved when flow is decelerated to rest adiabatically

SFEE

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - \cancel{q}^{\,0} + \cancel{W_s}$$

no shaft work

Neglect potential energy $\quad PE << KE$

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2$$

$$\boxed{h_{o1} = h_{o2}}$$

For isentropic flow of an ideal gas, stagnation enthalpy is constant

$$c_p T_{o1} = c_p T_{o2}$$

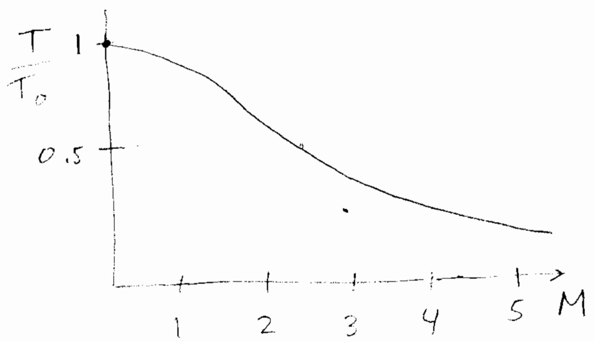
$$T_1 + \frac{V_1^2}{2c_p} = T_2 + \frac{V_2^2}{2c_p}$$

Rewrite the Relation between static + stagnation properties is a function of M.

$$c_p T_o = c_p T + \frac{1}{2}V^2$$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2C_pT}$$

$$\frac{T_0}{T} = 1 + \frac{(k-1)V^2}{2\,kRT} = 1 + \frac{k-1}{2}M^2$$



$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/k-1} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/k-1} = \left(1 + \frac{k-1}{2}M^2\right)^{1/k-1}$$

At $M = 1$ (sonic point) for <u>air</u>

$$\frac{T^*}{T_0} = \frac{2}{k+1} = 0.833$$

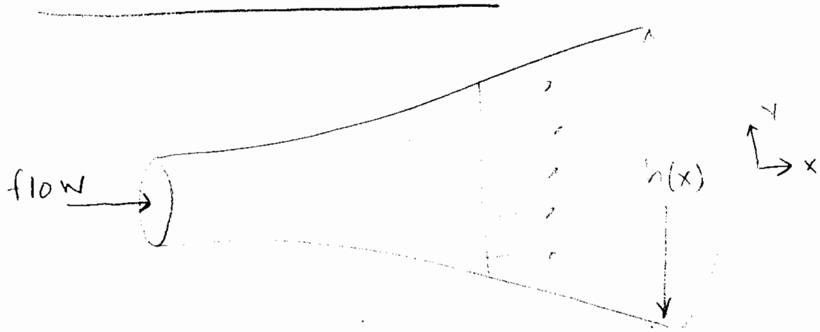
etc.

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/k-1} = 0.5283$$

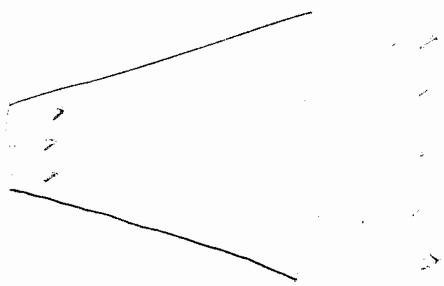
$*$ indicates property at $M = 1$ $(A^*, V^*, \rho^*, P^*, T^*)$

Variable Area Duct

Quasi-1D flow if $\frac{dh}{dx} \ll 1$



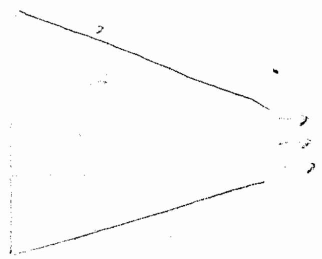
II. Supersonic flow ($M > 1$)



for $dA > 0$ huh? $dp < 0$

$dV > 0$

supersonic nozzle



for $dA < 0$ $dp > 0$

$dV < 0$

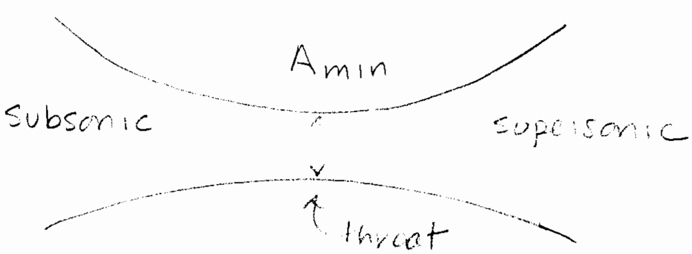
supersonic diffuser

Questions:

1) Rearrange or use momentum equation $\frac{dp}{\rho} + VdV = 0$ and determine whether $dp < 0$ or $dp > 0$ for each case.

2) What condition has to be true to have sonic conditions?

$dA = 0$, min. or max. duct area



can smoothly accelerate a flow from subsonic to supersonic through a throat

Converging Diverging nozzle required to accelerate flow from subsonic to sonic conditions

Initially $P_e = P_o$

Vacuum pump

Continuity

$\dot{m}$ = constant

$\rho V A$ = constant

$$d(\rho \cdot A) = 0$$

Chain rule : $VA\,d\rho + \rho A\,dV + \rho V\,dA = 0$

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad (1.)$$

Momentum :

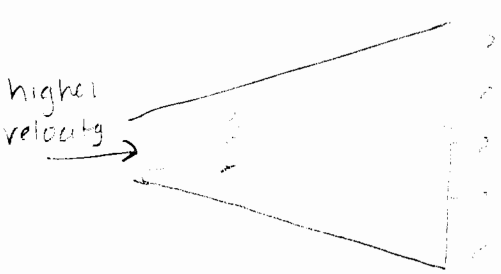
$$\frac{dp}{\rho} + V\,dV = 0 \qquad (2)$$

Speed of sound :

$$\frac{dp}{d\rho} = c^2 \qquad (3)$$

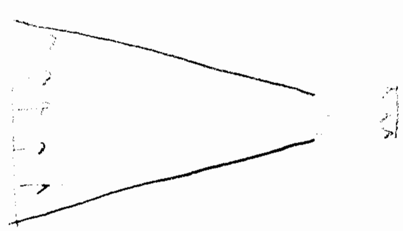
$$\frac{dV}{V} = \frac{dA}{A} \frac{1}{M^2 - 1}$$

I. Subsonic flow $(M < 1)$



for $dA > 0$, $dV < 1$, $dp > 0$

Subsonic diffuser
-decelerating flow



for $dA < 0$, $dV > 1$, $dp < 0$

Subsonic nozzle
-accelerating flow