Exam 2 Review

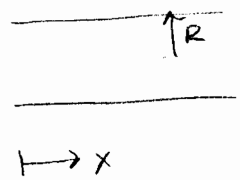
Viscous Flow in pipes
Compressible Flow
Conduction Heat Transfer

take into exam:

2 pages cheat sheets
compressible flow tables

Viscous flow in pipes

Poiseuille flow



$$u(r) = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right)R^2\left(1-\frac{r^2}{R^2}\right)$$

$$\bar{u} = \frac{R^2}{8\mu}\left(-\frac{dp}{dx}\right)$$

$$u(r) = 2\bar{u}\left(1 - r^2/R^2\right)$$

can rewrite these equations in terms of diameter, D

remember that $\frac{dp}{dx} \approx \frac{\Delta P}{L}$

$$\tau_w = \mu \left.\frac{\partial u}{\partial r}\right|_{r=R} = \frac{R}{2}\left(-\frac{dp}{dz}\right)$$

$D_h = \frac{4A_c}{P}$ for non-circular ducts

Velocity entry lengths (laminar + turbulent flows)

Steady Flow Energy Equation (in terms of head)

$$\frac{P_1}{\rho g} + \alpha\frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha\frac{V_2^2}{2g} + z_2 - \underset{\substack{\uparrow \\ \text{pump} \\ \text{work}}}{h_p} + \underset{\substack{\uparrow \\ \text{head loss} \\ \text{(major losses)}}}{h_f} + \underset{\substack{\uparrow \\ \text{minor} \\ \text{losses}}}{\sum h_m}$$

$$\Sigma h_m = \frac{V^2}{2g} \Sigma K$$

↑ loss coefficients

Compressible Flow

Ranges

	$M < 0.3$	subsonic, incompressible
	$M > 0.3,\ M < 1$	subsonic, compressible
	⋮	
	. and so on	

Properties of air

$k = 1.4$

$R = 287\ J/kg\,K$

$\rho = P/RT$

$c = \sqrt{kRT}$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{k/k-1}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + [(k-1)/2] M^2}{(k-1)/2}\right]^{\frac{1}{2}(k+1)/(k-1)}$$

or use tables

$$\dot{m}_{max} = 0.6847\, A^* \rho_0 (RT_0)^{1/2} = \frac{0.6847\, P_0 A^*}{(RT_0)^{1/2}}$$

Conduction Heat Transfer

$$\dot{Q} = qA = -KA\frac{dT}{dx}$$ Fourier's Law

$$\dot{Q} = hA(T_s - T_\infty)$$ Newton's Law of Cooling

$$\dot{Q} = \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$ Stefan-Boltzmann Law

Plane slab

$$\frac{d^2T}{dx^2} = 0 \rightarrow T = ax + b$$

Possible BC's $T = \text{constant}$

$$-K\frac{dT}{dx}\Big|_{x=0} = q$$ = constant surface heat flux

$$\frac{dT}{dx}\Big|_{x=0} = 0$$ → adiabatic or insulated

$$-K\frac{dT}{dx}\Big|_{x=0} = h(T_s - T_\infty)$$

Thermal resistance $R_{cond} = \frac{L}{KA}$

$R_{conv} = \frac{1}{hA}$

Contact Resistance $R_{t,c}$ [K/W]

Internal Heat Generation

$$\frac{d^2T}{dx^2} + \frac{q'''}{K} = 0$$

$$\rightarrow T = \frac{-q'''}{2K}x^2 + C_1x + C_2$$

Radial Systems

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \rightarrow T = a\ln r + b$$

Thermal resistance $R_{cond} = \frac{\ln(r_o/r_i)}{2\pi KL}$

$R_{conv} = \frac{1}{hA}$

Critical thickness of insulation $r = K/h$

Fins and extended surfaces

$$m^2 = \frac{hP}{KA_c} \qquad \theta = T - T_\infty$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Possible tip B.C.'s

- Constant temp. at tip
- Adiabatic / insulated tip
- Infinitely long fin
- Convection on tip

Insulated tip:

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

Look at table 3.4 for other sol'ns

$$Q = \sqrt{hPKA_c}\,\theta_b \tanh(mL)$$

$$\eta_f = \frac{\tanh mL}{mL}$$

Transient Heat Transfer

General Lumped Thermal Capacity Model

$$\underbrace{q A_s}_{\text{external heat flux}} + \underbrace{q''' \forall}_{\text{internal heat gen}} - \underbrace{q_{conv} A_s}_{\text{convective heat loss}} - \underbrace{q_{rad} A_s}_{\text{radiation heat los}} = \underbrace{\rho \forall c \frac{dT}{dt}}_{\text{change in internal energy}}$$

With convection only

$$-hA_s(T - T_\infty) = \rho \forall c \frac{dT}{dt}$$

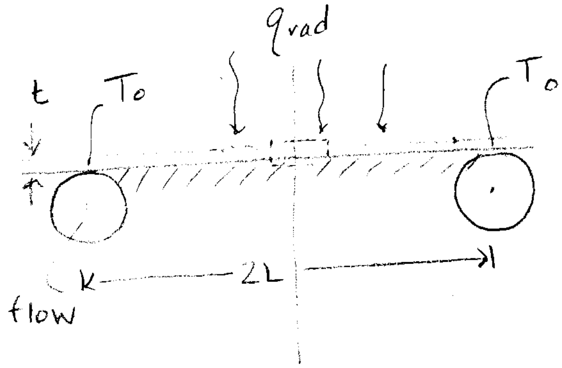
When to use?

$Bi < 0.1$

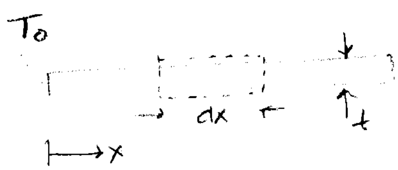
$$\frac{T - T_\infty}{T_i - T_\infty} = e^{\left(\frac{-hA}{\rho c \forall} t\right)}$$

Example: Extended Surface

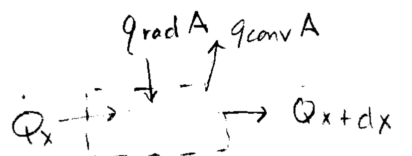
Solar collector plate h, T_∞



Note: Symmetry!



$$\left.\frac{dT}{dx}\right|_{x=L} = 0$$



$$\dot{Q}_x - \dot{Q}_{x+dx} + q_{rad}A - q_{conv}A = 0$$

$$\dot{Q}_x - \left(\dot{Q}_x + \frac{d\dot{Q}}{dx}\Delta x\right) + \varepsilon\sigma\left(T^4 - T_\infty^4\right)$$