2) ε - Ntu Method

ε = effectiveness of the heat exchanger = $\dfrac{Q_{actual}}{Q_{max}}$ $\quad 0 \le \varepsilon \le 1$

Ntu = number of (thermal) transfer units (also dimensionless) $\quad Ntu = \dfrac{UA}{C_{min}} \quad 0 \le Ntu$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{(\dot{m}c_p)_{min}}{(\dot{m}c_p)_{max}} \qquad 0 \le C_r \le 1$$

$$\varepsilon = f\,(Ntu,\ C_r)$$

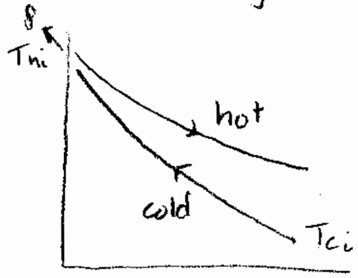
Analysis

$$\dot{m}_h c_{ph}(T_{hi} - T_{ho}) = \dot{m}_c c_{pc}(T_{co} - T_{ci})$$

$$C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci})$$

always true (+ useful!)

Infinite length HX

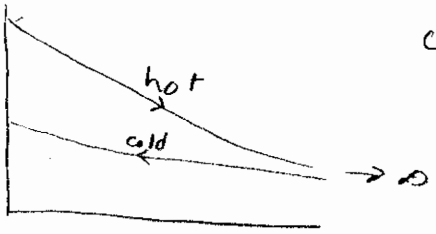


Define Qmax

Case ①

$$\left.\begin{matrix} C_h = C_{max} \\ C_c = C_{min} \end{matrix}\right\} (T_{hi} - T_{ho}) \le (T_{co} - T_{ci})$$

$$Q_{max} = C_{min}(T_{hi} - T_{ci})$$



case ②

$$\left.\begin{matrix} C_h = C_{min} \\ C_c = C_{max} \end{matrix}\right\} (T_{hi} - T_{ho}) \ge (T_{co} - T_{ci})$$

$$\dot{Q}_{max} = C_{min}(T_{hi} - T_{ci}) \rightarrow$$

$$\boxed{Q_{max} = C_{min}\,\Delta T_{max}}$$

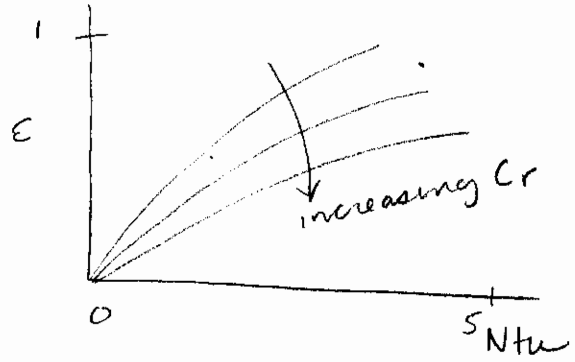
$$\varepsilon = \frac{Q_{actual}}{Q_{max}} = \frac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \frac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})}$$

Plot ε as a function of Ntu, C_r

If we have $\underbrace{\dot{m}\text{'s},\ c_p\text{'s}}_{C_{min},\ C_r}$, and UA → find $\varepsilon = \frac{Q_{act}}{Q_{max}}$

If we have 2 of 4 temps, use $\varepsilon = \frac{Q_{act}}{Q_{max}}$ to find 3rd temp,

then energy balance $(C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci}))$ to find 4th.



tables or figures
11.3 - 11.4

Ex : Single stream, parallel flow

$$\ln \frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \Rightarrow \varepsilon = \frac{1 - \exp\left[-Ntu(1+C_r)\right]}{1 + C_r}$$

Important Limits /Notes

① "Sizing" a heat exchanger?

Find Ntu → UA → A

(Solve for Ntu in equations or use figures)

③ Special limits $C_r = 1$ (matched flow) $C_c = C_h$

$C_r = 0$ $(C_{min} \ll C_{max})$ condenser evaporator

$$\varepsilon = 1 - \exp(-Ntu)$$

11.10 Water at a rate of 45,500 kg/h is heated from 80 to 150°C in a heat exchanger having two shell passes and eight tube passes with a total surface area of 925 m^2. Hot exhaust gases having approximately the same thermophysical properties as air enter at 350°C and exit at 175°C. Determine the overall heat transfer coefficient.

11.49 Saturated process steam at 1 atm is condensed in a shell-and-tube heat exchanger (one shell, two tube passes). Cooling water enters the tubes at 15°C with an average velocity of 3.5 m/s. The tubes are thin-walled and made of copper with a diameter of 14 mm and length of 0.5 m. The convective heat transfer coefficient for condensation on the outer surface of the tubes is 21,800 $W/m^2 \cdot K$.

(a) Find the number of tubes/pass required to condense 2.3 kg/s of steam.

(b) Find the outlet water temperature.

(c) Find the maximum possible condensation rate that could be achieved with this heat exchanger using the same water flow rate and inlet temperature.

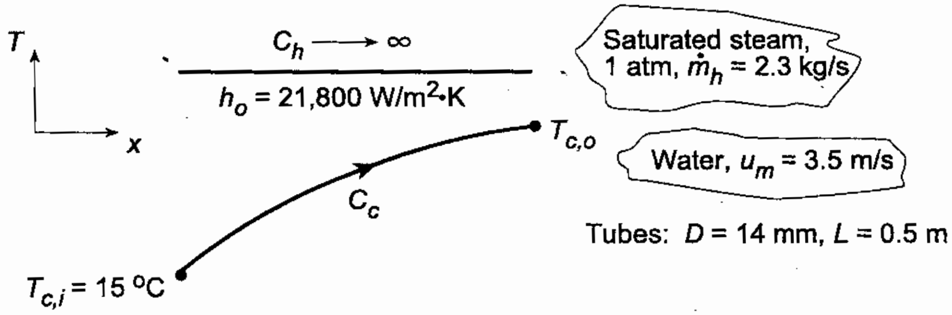
$\boxed{\text{(d)}}$ Using the heat transfer surface area found in part (a), plot the water outlet temperature and steam condensation rate for water mean velocities in the range from 1 to 5 m/s. Assume that the shell-side convection coefficient remains unchanged.

PROBLEM 11.49

KNOWN: Shell(1)-and-tube (two passes, $p = 2$) heat exchanger for condensing saturated steam at 1 atm. Inlet cooling water temperature and mean velocity. Thin-walled tube diameter and length prescribed, as well as, convective heat transfer coefficient on outer tube surface, h_o.

FIND: (a) Number of tubes/pass, N, required to condense 2.3 kg/s of steam, (b) Outlet water temperature, $T_{c,o}$, (c) Maximum condensation rate possible for same water flowrate and inlet temperature, and (d) Compute and plot $T_{c,o}$ and the condensation rate, $\dot{m}_h$, for water mean velocity, u_m, in the range $1 \le u_m \le 5$ ms/, using the heat transfer surface area found in part (a) assuming the shell-side convection coefficient remains unchanged.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to the tube walls.

PROPERTIES: *Table A.6*, Saturated steam (1 atm): $T_{sat} = 100°C$, $h_{fg} = 2257$ kJ/kg; Water (assume $T_{c,o} \approx 25°C$, $\overline{T}_m = (T_h + T_c)/2 \approx 295$ K): $\rho = 1/v_f = 998$ kg/m³, $c_c = c_{p,h} = 4181$ J/kg·K, $\mu = \mu_f = 959 \times 10^{-6}$ N·s/m², $k = k_f = 0.606$ W/m·K, $Pr = Pr_\ell = 6.62$.

ANALYSIS: (a) The heat transfer rate for the heat exchanger is

$$q = \dot{m}_h h_{fg} = 2.3\,\text{kg/s} \times 2257 \times 10^3\,\text{J/kg} = 5.191 \times 10^6\,\text{W} \tag{1}$$

Using the ε-NTU method, evaluate the following parameters:

Water-side heat transfer coefficient:

$$Re_D = \frac{u_m D}{\mu/\rho} = \frac{3.5\,\text{m/s} \times 0.014\,\text{m}}{959 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 / 998\,\text{kg/m}^3} = 50{,}993 \tag{2}$$

$$h_i = \frac{k}{D} Nu_D = \frac{k}{D} 0.023 Re_D^{0.8} Pr^{1/3} = \frac{0.606\,\text{W/m}\cdot\text{K}}{0.014\,\text{m}} \times 0.023 (50{,}993)^{0.8} (6.62)^{1/3} = 10{,}906\,\text{W/m}^2\cdot\text{K} \tag{3}$$

using the Colburn equation for fully developed turbulent conditions.

Overall coefficient:

$$\overline{U} = (1/h_i + 1/h_o)^{-1} = (1/10{,}906 + 1/21{,}800)^{-1} = 7269\,\text{W/m}^2\cdot\text{K} \tag{4}$$

Effectiveness relations: With $C_{min} = C_c$ and $\dot{m}_c = \rho(\pi D^2/4) u_m N$,

$$q = \varepsilon q_{max} = \varepsilon C_{min} \left(T_{h,i} - T_{c,i}\right) \tag{5}$$

$$C_{min} = \dot{m}_c c_c = 998\,\text{kg/m}^3 \left(\pi \times 0.014^2\,\text{m}^2/4\right) \times 3.5\,\text{m/s} \times N \times 4181\,\text{J/kg}\cdot\text{K} = 2248\,N \tag{6}$$

Continued...

$$5.191\times10^{6}\,\mathrm{W} = \varepsilon\times2248\,\mathrm{N}\,(100-15)\,\mathrm{K}$$

$$\varepsilon N = 27.17 \qquad (7)$$

From Eq. 11.36a with $C_r = 0$, the effectiveness is

$$\varepsilon = 1-\exp(-\mathrm{NTU}) = 1-\exp(-0.142) = 0.132 \qquad (8)$$

where, using $A_s = \pi DLNP$, NTU is evaluated as,

$$\mathrm{NTU} = \frac{\bar{U}A_s}{C_{min}} = \frac{7269\,\mathrm{W/m^2\cdot K}\,(\pi\times0.014\,\mathrm{m}\times0.5\,\mathrm{m})\,N\times2}{2248\,N} = 0.142$$

Hence, using Eq. (7), the required number of tubes is

$$N = 27.17/\varepsilon = 205.8 \approx 206$$ <

and the total surface area is

$$A_s = \pi DLNP = \pi\times0.014\,\mathrm{m}\times0.5\,\mathrm{m}\times206\times2 = 9.06\,\mathrm{m^2}.$$

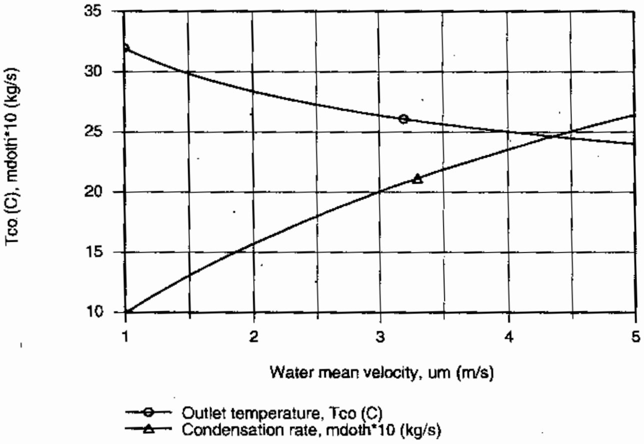
(b) The water outlet temperature with $C_{min} = 2248\,N = 463{,}090$ W/K,

$$T_{c,o} = T_{c,i} + q/C_{min} = 15^\circ\mathrm{C} + 5.191\times10^{6}\,\mathrm{W}/463{,}090\,\mathrm{W/K} = 26.1^\circ\mathrm{C}$$ <

(c) The maximum condensation rate will occur when $q = q_{max}$. Hence

$$\dot{m}_{h,max} = \frac{q_{max}}{h_{fg}} = \frac{C_{min}\left(T_{h,i}-T_{c,i}\right)}{h_{fg}} = \frac{463{,}090\,\mathrm{W/K}\,(100-15)\,\mathrm{K}}{2257\times10^{3}\,\mathrm{J/kg}} = 17.44\,\mathrm{kg/s}.$$ <

(d) Using the *IHT Heat Exchanger Tool, All Exchangers, $C_r = 0$,* along with the *Properties Tool* for *Water,* the foregoing analysis was performed to obtain $T_{h,o}$ and $\dot{m}_h$ using the heat transfer surface area $A_s = 9.06\,\mathrm{m^2}$ (part a) as a function of u_m.



Note that the condensation rate increases nearly linearly with the water mean velocity. The cold water outlet temperature decreases nearly linearly with u_m. We should expect this behavior from energy balance considerations. Since h_h is nearly two times greater than h_c, $\bar{U}$ is controlled by the water side coefficient. Hence $\bar{U}$ will increase with increasing u_m.

COMMENTS: Note that the assumed value for $\bar{T}_m$ to evaluate water properties in part (a) was a good choice.