Moment of Moment Equation: (Angular Momentum)

Moment of a force = torque

$$\frac{\partial}{\partial t}\int_{CV}(\vec{r}\times\vec{V})\rho\, d\forall + \int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\vec{n}\, dA = \sum(\vec{r}\times\vec{F})_{CV}$$

Energy Equation

1st Law of Thermodynamics

$$\delta E = \delta Q - \delta W$$

As a rate equation

$$\frac{DE_{sys}}{Dt} = \sum\dot{Q} - \sum\dot{W}$$

Sign conventions:

heat in $+$
heat out $-$
work in $-$
work out $+$

opposite from Munson + Young
same as thermo

Reynold's Transport Thm.

$$\frac{DE_{sys}}{Dt} = \frac{DE_{CV}}{Dt} = \frac{\partial}{\partial t}\int_{V}\rho e\, d\forall + \int_{A} e\rho\vec{V}\cdot\vec{n}\, dA$$

$$\boxed{\frac{\partial}{\partial t}\int_{V}\rho e\, d\forall + \int_{A} e\rho\vec{V}\cdot\vec{n}\, dA = (\dot{Q}_{net\ in} - \dot{W}_{net\ in})_{CV}}$$

Cons. of Energy

$\dot{Q}_{net\ in}$: conduction, convection, radiation

$\dot{W}_{net\ in}$: rotating shaft

$$e = \underset{\substack{\text{Int.}\\ f(T)}}{u} + \underset{\text{KE}}{\tfrac{1}{2}V^2} + \underset{\text{PE}}{gz}$$

Differential Forms of Conservation Equations

- learn something about flow field

→ Assumption: <u>Incompressible flow</u>

Review: Material Derivative of a fluid property B (scalar or vector)

$$\frac{DB}{Dt} = \frac{\partial B}{\partial t} + (\vec{V} \cdot \nabla) B \qquad \vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\frac{DB}{Dt} = \frac{\partial B}{\partial t} + u\frac{\partial B}{\partial x} + v\frac{\partial B}{\partial y} + w\frac{\partial B}{\partial z}$$

$$\frac{D\vec{B}}{Dt} = \frac{\partial \vec{B}}{\partial t} + (\vec{V} \cdot \nabla)\vec{B}$$

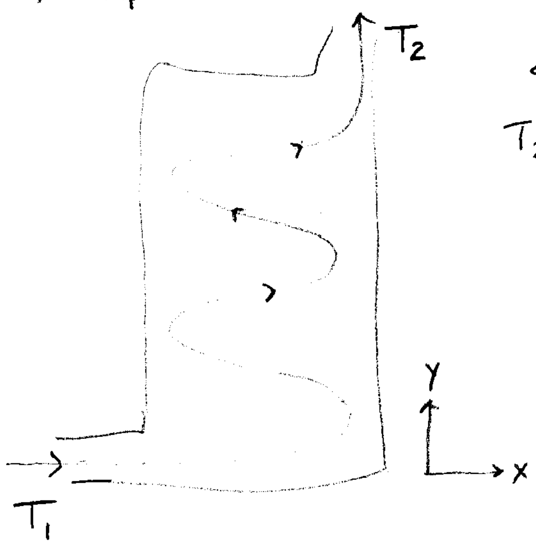
$$x: \quad \frac{\partial B_x}{\partial t} + u\frac{\partial B_x}{\partial x} + v\frac{\partial B_x}{\partial y} + w\frac{\partial B_x}{\partial z}$$

$$y: \quad \frac{\partial B_y}{\partial t} + u\frac{\partial B_y}{\partial x} + v\frac{\partial B_y}{\partial y} + w\frac{\partial B_y}{\partial z}$$

$$z: \quad \frac{\partial B_z}{\partial t} + u\frac{\partial B_z}{\partial x} + v\frac{\partial B_z}{\partial y} + w\frac{\partial B_z}{\partial z}$$

But what does it actually mean?

Example from book : Water heater



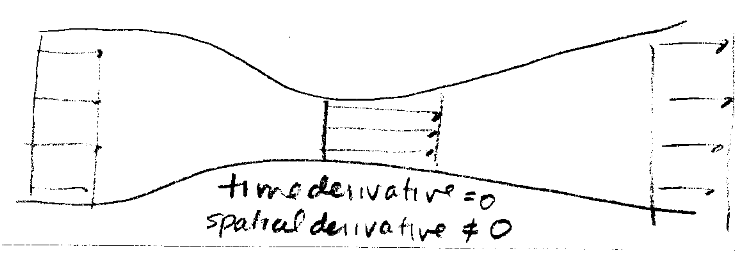
Steady flow

$T_2 > T_1$

$$\frac{DT}{Dt} \neq 0$$

$$\frac{\partial T}{\partial t} = 0$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \neq 0$$



Incompressible Flow

Mass: $$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0 \qquad \text{(all cases)}$$

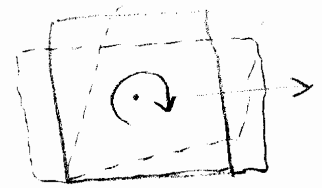
$$\frac{\partial \rho}{\partial t} + \underbrace{\cancel{\vec{V} \cdot \nabla \rho}}_{0,\ \text{incomp.}} + \rho \nabla \cdot \vec{V} = 0 \qquad \text{(incompressible)}$$

Momentum: $$\frac{\partial}{\partial t}(\rho \vec{V}) + (\vec{V} \cdot \nabla)(\rho \vec{V}) = \boxed{\text{Force terms}} \qquad \text{all cases}$$

$$\frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = \frac{1}{\rho}\left(\boxed{\text{Force terms}}\right) \qquad \text{incompressible}$$

What are force terms? Nextpage

Differential fluid element:



Translation → fluid acceleration

Rotation → vorticity, velocity gradient

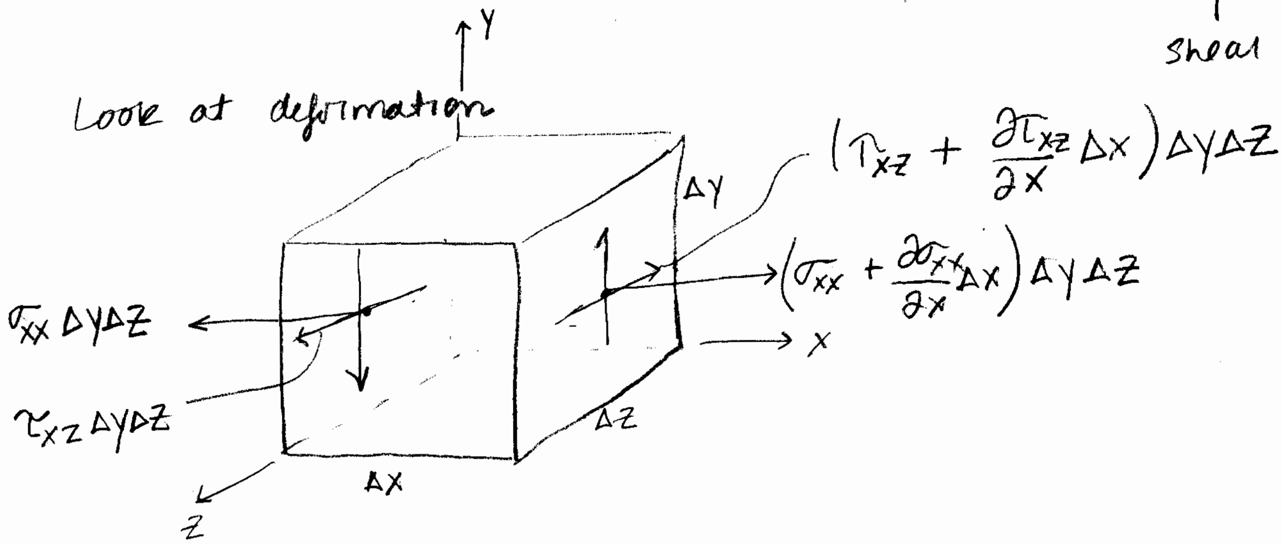
Deformation → normal + tangential forces

(pressure , viscous)

↑

shear stress

Look at deformation



1. Normal stress (could be pressure)
2. Tangential/shear stress (due to viscosity)

Sum all forces and get

$$\Sigma F_x = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\Delta X \Delta Y \Delta Z$$

$$\Sigma F_y = \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\Delta X \Delta Y \Delta Z$$

$$\Sigma F_z = \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)\Delta X \Delta Y \Delta Z$$

3. Body forces

$$\Sigma F_{x,b} = m g_x$$

$$\Sigma F_{y,b} = m g_y$$

$$\Sigma F_{z,b} = m g_z$$

Put these into Cons. of Mom. Eq.
+ write out components

$$x: \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g_x + \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

$$y: \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = g_y + \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)$$

$$z: \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = g_z + \frac{1}{\rho}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)$$

Differential Equations of Motion for a Fluid (incompressible)

Inviscid flow: (incompressible) $\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \vec{g}$ Euler's Eqn.

Along a streamline: $p + \frac{1}{2}\rho V^2 + gz = \text{constant}$ Bernoulli

Newtonian Fluid: (incompressible viscous steady/unsteady) viscous stresses (shear stresses) ~ viscosity

normal stresses (σ) = pressure + normal component of viscous stress

Using this relationship $\tau_{xy} = \mu \frac{\partial u}{\partial y}$ for all component, can substitute back into Eqns of Motion to get N-S Eq.

$$\boxed{\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla p + \vec{g} + \frac{\mu}{\rho}\nabla^2\vec{V}}$$ N-S Eqn.

$\nabla^2\vec{V}$ → laplacian

Write it out

$$x: \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g_x + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

etc ....