

Franklin W. Olin College of Engineering  
ENGR 3310: Transport Phenomena

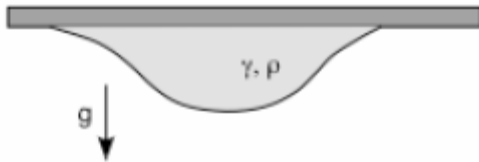
Problem Set 4

Assigned: 9/30/04  
Due: 10/7/04 by 5:00 pm

Fall 2004

**1. Pendant Drop**

a) Use dimensional analysis to estimate the maximum size of a pendant drop of fluid with surface tension,  $\sigma$ , and density,  $\rho$ .



b) If water is dripping from your ceiling, use part (a) and the material properties of water to estimate the size of the drops ( $\sigma_{water} = 70$  dynes/cm).

**2. Self-Similar Scaling of Winged Objects**

If various objects are geometrically self-similar then they may obey quite simple power-law relationships for their dynamic properties. A good example of this is the weight-velocity relationship of winged objects such as insects, birds and planes.

a) We defined in class the dimensionless lift coefficient  $C_L$  for a typical wing. Experiments show that for a typical wing of surface (planform) area  $S$  at a normal ‘angle of attack’ (about  $6^\circ$ ) the lift coefficient is  $C_L = 0.6$ . Use this, together with a steady-state force balance for an object (insect, bird or plane) flying at sea-level in air to show that the weight of the object and the steady cruising speed  $V$  are related by

$$W \approx 0.38SV^2 \quad (2.1)$$

b) If we now approximate all winged objects of some characteristic size (say their length)  $l$  as approximately geometrically self-similar, then we can express the surface area  $S$  and weight  $W$  as proportional to particular powers of  $l$ . (i.e. we write  $W \sim l^m$  where  $m$  is an exponent to be determined by you and ‘ $\sim$ ’ means ‘scales as’ or ‘is proportional to’) Write down these power-law relationships and use them to eliminate  $A$  from equation 2.1 in

favor of a scaling relationship between the weight  $W$  and the cruising velocity  $V$  *alone!*

Notice how strong your scaling is (i.e. how big the exponent *is!!*).

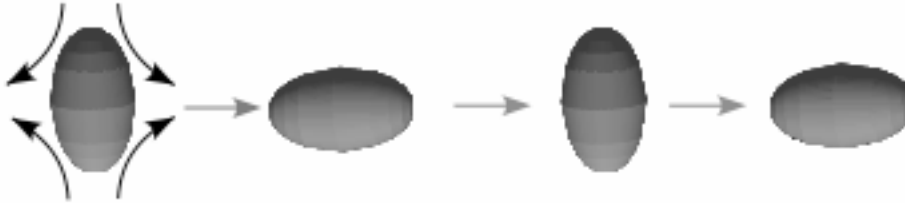
- c) Verify that your scaling is correct by plotting the following data (in Excel or Matlab) on a Log-Log plot and fitting a straight line to this data. Determine the coefficient in the scaling relationship (i.e. the constant  $C$  in a curve of form  $y = Cx^n$ ).

	Mass $m$	Wing Area $S$ [ $m^2$ ]	Cruising Speed $V$
Crane fly	30 mg	7.5e-5	3 m/s
Common starling	80 g	0.02	10.3 m/s
Canadian goose	5.7 kg	0.28	23 m/s
Cessna Citation	2.0 metric tons	18.2	120 mph
Boeing 747	350 metric ton	511.0	570 mph

- d) Aeronautical design engineers are often interested in another parameter, the *Wing Loading* defined by  $wl = W/S$  since this is a measure of the stress acting on the wings. Rearrange your scaling relationship (and/or combine it with equation (2.1)) to express your answer as a new power-law equation of the form  $W = A(wl)^p$  and give values for  $A$  and  $p$ . Assuming it falls on the curve you found in part (c), what is the wing-loading for a Boeing 767 with a weight of 250 metric tons and a cruising speed of 550 mph? Is this a large value (express your answer in  $N/m^2$  and also in psi)?
- e) Finally, if some of you have been to the Museum of Science in Boston you may have seen the Gossamer Condor; a human-powered ultra-light aircraft. It has a weight of 940 N, a wing area of  $S = 70 m^2$  and was designed for a cruising speed of 5 m/s (a comfortable cycling speed!). Add this data point to your plot of part (c)...why do you think this plane's design parameters put it where it is...what would the consequences be if it was designed to fall on same curve as other winged objects?

### 3. Oscillating Drops

A small drop of water in space (i.e.  $g=0$ ) is perturbed from a spherical shape. The resulting oblate shape oscillates at a frequency,  $f$ , as shown in the figure:



- What physical effect do you expect will be important in this system (Inertia? Gravity? Etc.?)
- Calculate the relevant Pi groups for the drop.
- What are the relevant Pi groups in the limit  $Re \gg 1$  (there should only be one!)? Use this Pi group to find how the frequency scales with the density,  $\rho$ , and the radius,  $R$ .
- Now consider instead of a small drop a water, a gigantic sphere of oscillating gas (i.e. a star). What are the relevant Pi groups for this system? (Hint: the units on the gravitational constant,  $G$ , are  $[L^3 M^{-1} T^{-2}]$ ). How does this frequency scale in the limit of  $Re \gg 1$ ?
- Sketch a log-log plot of  $f$  vs  $\rho$  and  $f$  vs  $R$  for both systems.

### 4. Dynamic Similarity in the Operation of Centrifugal pumps

The basic goal of a centrifugal pump is to increase the ‘available head’ or pressure in a pipe. This pressure increase  $\Delta p$  can then be used to pump fluid through a pipe network. As an engineer, a common task is to be asked to ‘size a pump’ *i.e.* specify the particular model number or size (characterized typically by the impeller diameter  $D$ ) needed to fulfill a particular delivery rate. The design charts typically provided for a series of pumps are based on dimensional analysis.

- For a particular series of geometrically-similar centrifugal pumps (i.e. a set of pumps all of the same design but of different sizes – so that they can all be characterized by a

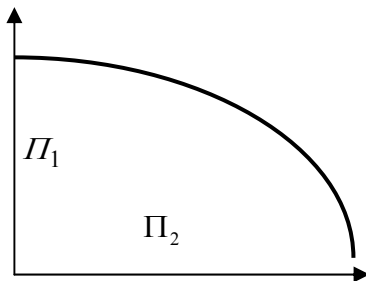
single length scale  $D$ ), the pressure rise  $\Delta p$  generated by the pump is a function of the following variables:

$$\Delta p = f(D, \rho, Q, \omega) \quad (1)$$

Here  $D$  is the impeller diameter,  $\omega$  is the angular velocity of the impeller,  $\rho$  is the fluid density and  $Q$  is the volume flow rate through the pump.

(Note: The fluid viscosity (and the resulting viscous stress) is typically unimportant in high-speed centrifugal pumps; hence its absence from eq. (1).

Perform a dimensional analysis of this problem ensuring that the pressure drop and the volume flow rate  $Q$  are NOT repeated variables and that each appear only once in your resulting Pi groups. Write out the expected dimensionless form of the governing equation.



(b) For a particular class of centrifugal pumps, you are supplied by the manufacturer with a dimensionless plot that shows a dimensionless variable  $\Pi_1$  containing the pressure increase plotted vs. a variable  $\Pi_2$  containing the volume flow rate. The resulting curve is sketched at left and is given by the equation:

$$\Pi_1 = 0.064 - 0.001(\Pi_2)^2 \quad (2)$$

Use this design curve to answer the following questions:

1. If a  $D = 20$  cm pump is operated at a fixed angular speed of  $30\pi$  rad/s (900 rpm), what is the maximum pressure increase that can be achieved by the pump, assuming the fluid pumped is water? At what volume throughput  $Q$  does the increase in head provided by the pump drop to zero?
2. What should be the minimum diameter pump you specify to be purchased if you require at least  $1 \text{ m}^3$  of water per second to be delivered at a pressure of 10 psig. and the motor running the pump operates at a fixed angular speed of  $40\pi$  rad/s (1200 rpm). Pumps in this series come in impeller sizes of 8, 12, 16, 20, 24....cm.

(Specifying too small a pump will of course cost you a promotion, specifying an over large one will impact company profits and also your end-of-year bonus!!)

$$\text{Recall } 14.7 \text{ psi} = 1.01 \times 10^5 \text{ Pa.}$$

⇒ Hint as an initial guess try a particular size (say 20 cm...) and then iterate from there.

c) Another useful pump design parameter that does not involve the impeller diameter is known as the specific speed,  $N_s$ , which is also a dimensionless number. Create a third dimensionless group by taking the ratio of  $\Pi_1^a$  and  $\Pi_2^b$  and determine the exponents  $a$  and  $b$  such that the diameter cancels out. Remember that a dimensionless speed should have the actual speed,  $\omega$ , in the numerator.